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Iterative Learning Control for Biped Walking

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***Abstract*—In this paper, an iterative learning control (ILC) approach is proposed for biped walking control. The biped robot is powered by applying an impulsive push along the stance leg just before the heel strikes. The ILC law is designed based on Poincaré map, and applied to learn the desired impulsive push at every step in the presence of system uncertainties. The convergence of the proposed ILC approach for biped walking control is proofed, and the simulation results show that the proposed ILC approach for biped walking control can track the desired step length effectively, even if the mass of foot can not be ignored compared to that of pelvis.**

I. INTRODUCTION

RECENT years, many studies have been done for realizing dynamic walking of biped robots. The available control method which is used to generate a stable walking gait is the Zero Moment Point (ZMP) method [1]. The joint trajectories are generated according to the ZMP criteria, and then the biped robot's joints are controlled to follow the trajectories by servo motors. Many biped robots have been developed by the researchers, thereinto, the Humanoid robots developed in Japan should be the state-of-the-art bipedal robots [2,3], they can perform many complex motions and keep themselves stable. For present humanoid robots, the remaining key problem is the effectiveness of the humanoid robots which is limited by the amount of energy they requires. It is estimated that ASIMO′s energy consumption is some 32 times greater than that of a typical human [4]. The inefficiency of energy is due to the fact that the control method of humanoid robots originates from traditional industrial robot technology, and electrical drives with high gain PD controllers are often used, which make the joints stiff [5]. Passive Dynamic Walking (PDW) has been established that a suitably designed unpowered biped robot can walk down a gentle slope utilizing only gravity effect and generates a stable periodic gait [6]. The passive walker utilizes its physical dynamics and creates energy-effective walking pattern automatically. In order to walk on the level ground and to keep energy-effective, active walkers based on the passive dynamic walking are studied by many researchers [4,7]. The simplest PDW model has been extended with the addition of two types of actuations [7]. One is to apply an impulsive push along the stance leg just before the heel strikes in order to minimize energy consumption. The other method is to apply a hip torque on the stance leg using the torso as a base. The analytical results show that performing a toe-off push on the stance leg uses only a quarter of the energy that performing torque at the hip consumes [7].

The simplest walking model is limited as the ratio of the mass of each foot, *m*, to that of the pelvis, *M*, approaches to zero, and the motion of the stance leg is not affected by the swing leg [7,8]. Simple linearized equations are used to analyze the periodic cycle, and to determine the relationship between initial state of the robot and other parameters when the limit cycles appear. Because the relationships are obtained by the linearized model, a control mechanism must be designed when this method is applied to control a complex robot walking.

An ILC for biped walking control is proposed in this paper. The ILC law is designed based on Poincaré map. The only actuation is the toe-off push before the heel strikes, and the robot is set to be passive during the swing phase. The ILC law for toe-off push impulse is designed to track the desired walking step length. The convergence of the proposed ILC for biped walking control is proofed, and the simulation results show that the proposed ILC for biped walking control can track the desired step length effectively, even if the mass of foot can not be ignored compared to that of pelvis.

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II. THE BIPED ROBOT MODEL

A. Dynamic Model of the Swing Phase

The simplest model of a biped walking robot has two rigid legs, one joint is at the hip, and the other joint is at the point where the support leg touches the ground, a spring-like hip torque applied to the swing leg. The model is shown in Fig. 1. The dynamic equation of the robot in the swing phase can be obtained by Lagrange equation as follows:

$$[M+2m(1-\cos\phi)]l^2\ddot{\theta}+ml^2[(\cos\phi-1)\ddot{\phi}+\sin\phi(2\dot{\theta}\dot{\phi}-\dot{\phi}^2)] \\ -Mgl\sin\theta-mgl[\sin\theta+\sin(\phi-\theta)]=0 \quad (1)$$

$$ml^2[(\cos\phi-1)\ddot{\theta}+\ddot{\phi}-\dot{\theta}^2\sin\phi]+mgl\sin(\phi-\theta)=-k\phi \quad (2)$$

Where m is the mass of the foot, M is the mass of the pelvis, l is the length of the leg and g is the acceleration due to gravity. θ is the angle of the stance leg with respect to the ground normal. ϕ is the angle between the stance leg and the swing leg.

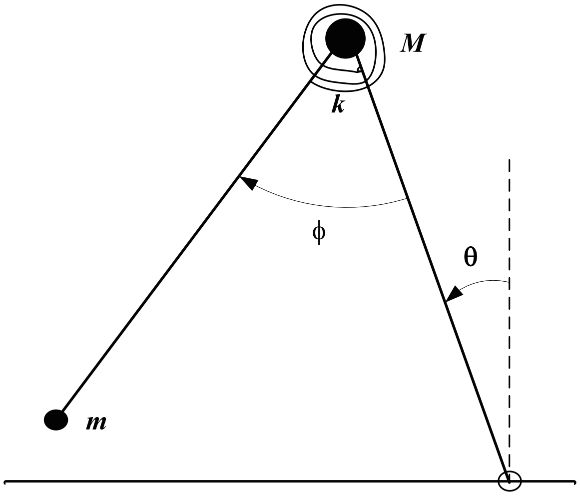

Figure 1. Diagram of the simplest biped robot.

k is the stiffness of spring at the hip joint.

Similar to the method described in [7- 8], the equations (1) and (2) can be expressed in dimensionless terms, using the pelvis M, leg length l, and time $t \triangleq \sqrt{l/g}$ as base units. Velocities will therefore be made dimensionless by factor $\sqrt{gl}$, the equations (1) and (2) are simplified as follows

$$\begin{aligned}&[1+2\rho(1-\cos\phi)]\ddot{\theta}+\rho(\cos\phi-1)\ddot{\phi}+\rho\sin\phi(2\dot{\theta}\dot{\phi}-\dot{\phi}^2)\\&-\sin\theta-\rho[\sin\theta+\sin(\phi-\theta)]=0\end{aligned} \tag{3}$$

$$(\cos\phi-1)\ddot{\theta}+\ddot{\phi}-\dot{\theta}^2\sin\phi+\sin(\phi-\theta)=-K\phi \tag{4}$$

Where $\rho = m/M$ is the ratio of mass, $K = k/mgl$ is the dimensionless torsional spring constant. Let $\rho \to 0$, the linearized approximations of (3) and (4) are

$$\ddot{\theta}-\sin\theta=0 \tag{5}$$

$$\ddot{\phi}-\ddot{\theta}+\omega^2\phi=0 \tag{6}$$

where ω is defined as

$$\omega=\sqrt{K+1} \tag{7}$$

B. Ground Impact

The boundary conditions for the swing leg can be determined according to that of the stance leg. With initial values $\theta(0)=\alpha$ and $\dot{\theta}(0)=\mathbf{\Omega}$, differential equations (5) and (6) can be integrated in time domain and ends at heel strike . At this time, the relationship between the angle ϕ and the angle θ is given as follows [7,8]

$$\phi(\tau)=2\theta(\tau) \tag{8}$$

Where τ is the step period. The collision with ground is assumed to be instantaneous and perfectly inelastic. The state following impact can be found using conservation of swing leg angular momentum about the hip and impulse–momentum equation of the whole robot about the swing leg contact point. The toe-off impulse P is applied to the stance leg just before the heel strikes, the jump conditions are

$$\theta^+=-\theta^-,\ \phi^+=-2\theta^- \tag{9}$$

$$\dot{\theta}^+=-\dot{\theta}^-\cos2\alpha+p\sin2\alpha,\ \dot{\phi}^+=\dot{\theta}^+(1-\cos2\alpha) \tag{10}$$

Where the superscript '+' means 'just after heel-strike', the superscript '-' means 'just before heel-strike'. The impulse P is dimensionless with normalization factor $M\sqrt{gl}$.

C. Finding Limit Cycles and Step Periods

In order to make the walking cycle be periodic, it is necessary for the composition of the preceding equations to yield a new set of initial conditions for a following step that are equal to the original conditions. The boundary conditions are [7]

$$\theta^+(\tau)=\theta(0)=\alpha,\ \dot{\theta}^+(\tau)=\dot{\theta}(0)=\mathbf{\Omega} \tag{11}$$

Considering only toe-off impulse is performed along the stance leg, analysis of limit cycles can be carried out using the linearized motion equations (5) and (6), with the jump conditions (9) and (10). Combining these equations with the boundary conditions (8) and (11), similar to the method described in [7- 8], we can conclude that the parameters, α, Ω, τ, ω and P must satisfy the following equations when the limit cycle exists.

$$\begin{aligned}&[\omega(2\omega^2+1)(e^\tau-1)\cos(\omega\tau/2)\\&+(e^\tau+1)\sin(\omega\tau/2)]\cos(\omega\tau/2)=0\end{aligned} \tag{12}$$

$$(e^\tau+1)\alpha=-(e^\tau-1)\mathbf{\Omega} \tag{13}$$

$$\begin{aligned}&\alpha(e^\tau-e^{-\tau})\cos2\alpha-2P\sin2\alpha\\&=\mathbf{\Omega}[2-(e^\tau+e^{-\tau})\cos2\alpha]\end{aligned} \tag{14}$$

If the natural frequency ω is predetermined, the step period τ can be obtained by solving the roots of (12). Obviously, $\tau = \pi/\omega$ is a root of (12), which is called 'short period' , and the limit cycle corresponding to this solution is unstable for PDW[8]. The set of roots associated with the '[]' term in (12) can be solved by numerical method. There are infinitely many solutions in (12), these larger-period roots correspond to

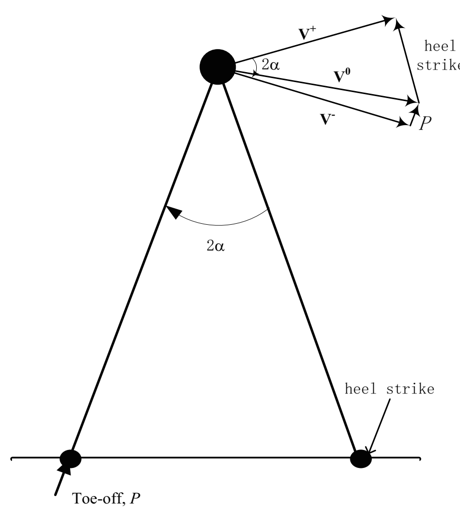

Fig. 2. Ground collision model.

multiple oscillations of the swing leg between heel-strikes. So in this paper, we are only interested in the smallest root with the '[]' term in (12).

Supposing that the linearized model is perfected and no disturbance exists, for a predetermined natural frequency ω, the step period τ can be determined by (12). If the desired walking length α is predetermined, the initial velocity Ω and the toe-off impulse P can be obtained by (13) and (14) respectively. Therefore, the control system design is very simple.

III. Iterative Learning for Biped Walking Control

A. Poincaré Map

If the complex model (1) – (4) is considered, and the mass ratio ρ doesn't tend to zero, the analytic expressions of the solution can not be obtained. It is difficult to find a toe-off impulse P to let the robot walk at a desired walking step length. Poincaré map is a powerful tool to analyze the limit cycle of the robot walking trajectories. The Poincaré section is selected at the start of a step, just after heel-strike, and the angular values of the stance leg and the swing leg satisfy (8). The Poincaré map is the map from one section to the next section, it can be described as follows

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}_k, P) \tag{15}$$

where $\boldsymbol{x}_k = [\theta\ \phi\ \dot{\theta}\ \dot{\phi}]_k^T$ is the initial value of state vector at step k. A period-one gait limit cycle of the robot walking trajectories can be described with Poincaré map

$$\boldsymbol{x}^* = \boldsymbol{f}(\boldsymbol{x}^*, P) \tag{16}$$

For the simplest walking model, the period-one gait limit cycle of the robot walking trajectories can be described by the angle of the stance leg α alone. Because the other states can be expressed by α (see (8), (10) and (13))

$$\alpha^* = g(\alpha^*, P) \tag{17}$$

Simulation results show that the similar relationships exist in the complex nonlinear biped robot model, and the limit cycle is influenced by toe-off impulse [7].

From the point of view of energy saving, we do not want to continuously apply control signals if the desired walking states can be reached with an appropriate toe-off impulse P. The biped robot periodic walking is repeated from one step to next step, and it can be described by the Poincaré map (15)-(17). In this paper, we are focused on walking step length control. The step length is dependent on the angle of stance leg α and the leg length l, when a robot has been constructed, the leg length l is invariable. So the control law is designed to track the angle of stance leg α only.

B. Iterative Learning Control

Iterative learning control (ILC) is a suitable method for dealing with repeated control tasks, and the exact models are not required to design the control law [9, 10]. In [10], the final state control in motion systems is addressed, and an initial state ILC approach is proposed for final state control of motion systems. The biped robot periodic walking is repeated from one step to next step. Inspired by the results of final state control of motion systems, the ILC technology is extended to tracking control of the desired angle of the stance leg α.

Because the stable regions are large enough for the biped robot actuated by toe-off impulse and installed a passive spring at the hip joint(see the Fig.4 in [7]), we assumed that the desired limit cycles are stable in the following discuss.

Assumption 1. Poincaré map $g(\alpha,P)$ is a continuous differentiable function on R^2.

Let α_d denote the desired initial angle of stance leg, and let P_d denote the toe-off impulse corresponding to the desired initial angle α_d.

For $\alpha_k \in D=\{\alpha \,|\, |\alpha-\alpha_d| < \delta, \delta>0 \text{ and } \alpha>0\}$, let P_k denote the toe-off impulse corresponding to α_k. expanding Poincaré map $g(\alpha_k,P_k)$ at (α_d,P_d) using Taylor's series ,we have

$$\begin{aligned} g(\alpha_k, P_k) = g(\alpha_d, P_d) + g'_\alpha(\alpha, P_k)(\alpha_k - \alpha_d) \\ + g'_P(\alpha_k, P)(P_k - P_d) \end{aligned} \tag{18}$$

Where $\alpha\in(\alpha_d,\alpha_k)$ and $P\in(P_d, P_k)$. From (18) we have

$$\begin{aligned} \alpha_d - \alpha_k &= g(\alpha_d, P_d) - g(\alpha_k, P_k) \\ &= -g'_\alpha(\alpha_k - \alpha_d) - g'_P(P_k - P_d) \end{aligned} \tag{19}$$

Combining (19) with Assumption 1, we have

$$|\alpha_d - \alpha_k| \le \lambda |P_k - P_d| \tag{20}$$

Where $\lambda = \max_{\alpha\in D} |g'_P/(1-g'_\alpha)|$.

Assumption 2. The α is a monotone function of P, such that if $P_i>P_j$ then $\alpha_i>\alpha_j$. Where the α_i is the stable step length and P_i is the toe-off impulse corresponding to the α_i.

The assumption 2 shows that the stable step length is increasing with the toe-off impulse increasing. This has been verified by simulation results (see Fig.3 (a) and (c) in [7]).

Let P_k denote the toe-off impulse at the kth iteration, where k = 1, 2, . . . denotes the iteration number. The ILC is

$$P_{k+1} = P_k + \gamma(\alpha_d - \alpha_k) \tag{21}$$

Where $\gamma > 0$ is a learning gain. To achieve learning convergence, a key issue is to determine the range of the learning gain γ, which is summarized in the following Theorem.

Theorem. Suppose that there exists an $M < \infty$ such that $|P_d - P_1| = M$. For any given $\varepsilon > 0$, applying the control law (21) and choosing the learning gain in the following range

$$(1-\rho)/\lambda < \gamma < (1+\rho)/\lambda,\ 0<\rho<1. \tag{22}$$

The ILC convergence is guaranteed if the learning gain is chosen to meet condition (22).

Proof. According to (20), there exists a quantity $0<\lambda_k\le\lambda$, such that

$$|\alpha_d - \alpha_k| = \lambda_k |P_k - P_d| \tag{23}$$

Combining Assumption 2, (20), (21) with (23), we have

$$
\begin{aligned}
\left|P_d - P_{k+1}\right| &= \left|(P_d - P_k) - \gamma(\alpha_d - \alpha_k)\right| \\
&= \left\|P_d - P_k\right| - \gamma\left|\alpha_d - \alpha_k\right\| = \left|1 - \gamma\lambda_k\right|\left|P_d - P_k\right|
\end{aligned}
\tag{24}
$$

The remains of the proof are similar to that in [10], so it is omitted in this paper (see Appendix B in [10] for detail).

IV. Simulation Results

In the simulation, the ω is selected first, then the root of (12) is searched by Newton iteration algorithm to obtain the walking step period. Second, the desired angle α_d is appointed, then the angular velocity Ω and toe-push impulse P is calculated by (13) and (14). The nonlinear differential equations (3) and (4) are integrated using standard Runge-Kutta method (RK4) with the boundary conditions (8) and (11). The integration is stopped when the swing leg angle is twice as large as the stance leg angle. Fig.3 shows the curve of the stance leg angle after every heel-strike. The mass ratio, initial stance leg angle and natural frequency are set as ρ=0.01, α=π/12≈0.2618 and ω=2 respectively. The step period obtained by numerical calculating is τ =1.6525, and the toe-off impulse is P=0.1034. The stance leg angle is stable at about 0.2630, and the real step period in simulation is stable at about 1.6319. The deviation is mainly caused by the nonlinear terms in the model (3) and (4). So a control law must be designed to realize the accurate tracking of the desired angle.

From Fig.3 we can find that several steps are needed before the robot settles down to the stable walking stage with an invariable toe-off impulse P. So the control effect of toe-off impulse P can not be evaluated simply by the robot's state after the impulse is applied. The ILC is modified as follows:

1. Apply a constant toe-off impulse P in a predetermined step interval. This value is set as 10 in the simulation.
2. Estimate the average walking step length by the walking step lengths during the interval.
3. Replace α_k with the average walking step length and update the toe-off impulse P by (21).

Fig.4 shows the curves of the stance leg angle after every heel-strike with two different desired angles. The mass ratio,

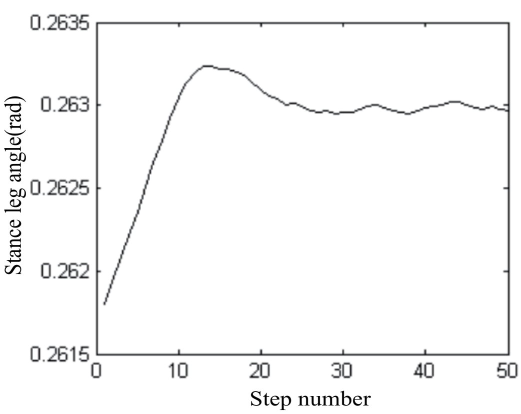

Fig. 3. Stance leg angle after every heel strike, ρ=0.01, ω=2.

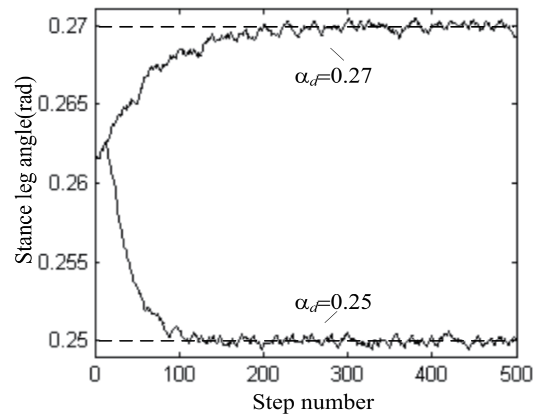

Fig. 4. Stance leg angle after every heel strike, ρ=0.01, ω=2. The desired angles are set as α_d=0.25, and α_d=0.27 respectively. The learning gain for ILC is γ=0.2.

initial stance leg angle and natural frequency are set as ρ=0.01, α=0.2618 and ω=2 respectively. The desired angles are set as α_d=0.25 and α_d=0.27 respectively. The learning gain for ILC is γ=0.2. From Fig.4 we can find that the stance leg angle converges to the desired angle α_d=0.25 after 10 iteration learning (100 steps) and converges to the desired angle α_d=0.27 after 20 iteration learning (200 steps).

For a real robot, the mass ratio generally does not tend to zero. Therefore, the simplified nonlinear models in [7] and [8] are unsuitable for this case. Fig.5 shows the curves of the stance leg angle after every heel-strike with two different mass ratios. The initial stance leg angle and natural frequency are set as α=0.2618 and ω=2 respectively. The desired angle is set as α_d=0.15, the mass ratios are set as ρ=0.1 and ρ=0.4 respectively. The learning gain for ILC is γ=0.2. From Fig.5 we can find that the stance leg angle converges to the desired angle α_d=0.15 after a few iteration learning (about 100 steps). Fig. 6 shows the learning process of toe-off impulse P, which converges to a constant value after 200 steps. Fig. 7 shows the phase space trajectories of the swing leg with the robot's

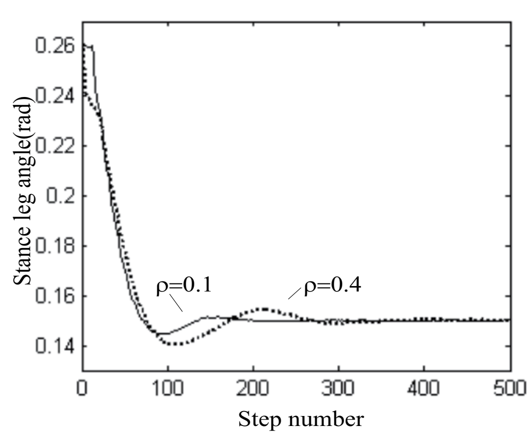

Fig. 5. Stance leg angle after every heel strike, α_d=0.15, ω=2. The mass ratios are set as, ρ=0.1 and ρ=0.4 respectively. The learning gain for ILC is γ=0.2.

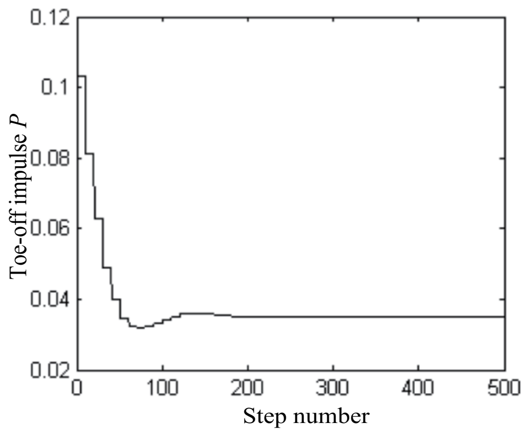

Fig. 6. Toe-off impulse at every heel strike, α_d=0.15, ω=2. The mass ratio is set as, ρ=0.1. The learning gain for ILC is γ=0.2.

walking process controlled by ILC, the mass ratio is ρ=0.4. Obviously, the trajectories asymptotically converge to a stable limit cycle (the dark region in the center of Fig.7). By contrast, using a constant toe-off impulse P designed by the simplified model, the trajectories are divergent, and stable gait can not be obtained.

V. Conclusion

The ILC is a suitable technology to learn the desired toe-off impulse for biped robot walking control, and the desired toe-off impulse can be obtained through step by step learning process despite the existence of unknown nonlinear uncertainties in the biped robot system. The proposed ILC approach for biped walking control can track the desired step length effectively, even if the mass of foot can not be ignored compared to that of pelvis.

The tasks in next phase are as follows:

1. Extend the proposed ILC approach to more generic biped robot.

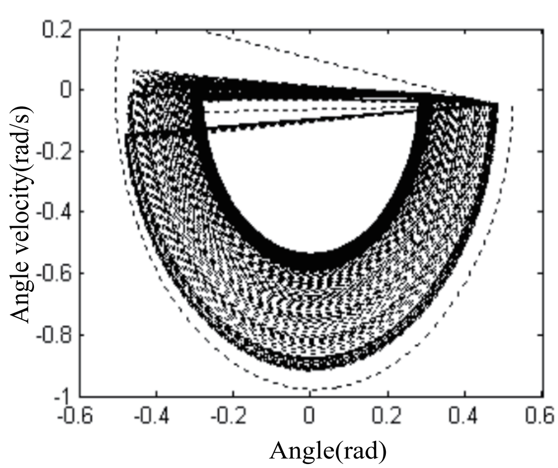

Fig. 7. The phase space trajectories of swing leg for the robot's walking process, α_d=0.15, ω=2. The mass ratio is set as ρ=0.4. The learning gain for ILC is γ=0.2. The robot walking processes are performed 500 steps continuously.

2. Let the swing leg be shortened in the swing phase to overcome the scuffing of the foot during mid-swing.

3. Design a biped robot to verify the effectiveness of the proposed learning approach by experiment.

References

[1] M.Vukobratovic, B. Borovac, "Zero-Moment point, thirty-five years of its Life," *International Journal of Humanoid Robot*, vol.1, pp.157–173, 2004.

[2] H Hirukawa, S. Kajita, F. Kanehiro, K. Kaneko, "The human-size humanoid robot that can walk, lie down and get up," *International Journal of Robotics Research*, vol.24, pp.755-769, 2005.

[3] K. Hirai, et al., "The development of Honda Humanoid Robot," *In Proc. IEEE Int. Conf. Robotics and Automation*, Leuven, Belgium, 1998, pp. 983–985.

[4] S.Collins, A. Ruina, R. Tedrake, and M. Wisse, "Efficient bipedal robots based on passive dynamic walkers," *Science*, vol. 307, pp.1082–1085, 2005.

[5] Vanderborght et al., "Comparison of Mechanical Design and Energy Consumption of Adaptable, Passive-compliant Actuators," *International Journal of Robotics Research*, vol.28, pp.90-103, 2009.

[6] T. McGeer, "Passive dynamic walking," *International Journal of Robotics Research*, vol.9, pp.62-82, 1990.

[7] A.D. Kuo, "Energetics of Actively Powered Locomotion Using the Simplest Walking Model," *J. Biomech. Eng*., vol.123, pp.264–269, 2001

[8] M. Garcia, A.Chatterjee, A. Ruina and M. Coleman, "The Simplest Walking Model: Stability, Complexity, and Scaling," *ASME J. Biomech. Eng.*, vol.120, pp. 281–288, 1998.

[9] J. X. Xu, "Recent Advances in Iterative Learning Control," *Acta Automatica Sinica*, vol.31, pp. 132-142, 2005

[10] J. X. Xu and D. Q. Huang, "Initial state iterative learning for final state control in motion systems," *Automatica*, vol.44, pp.3162–3169, 2008.